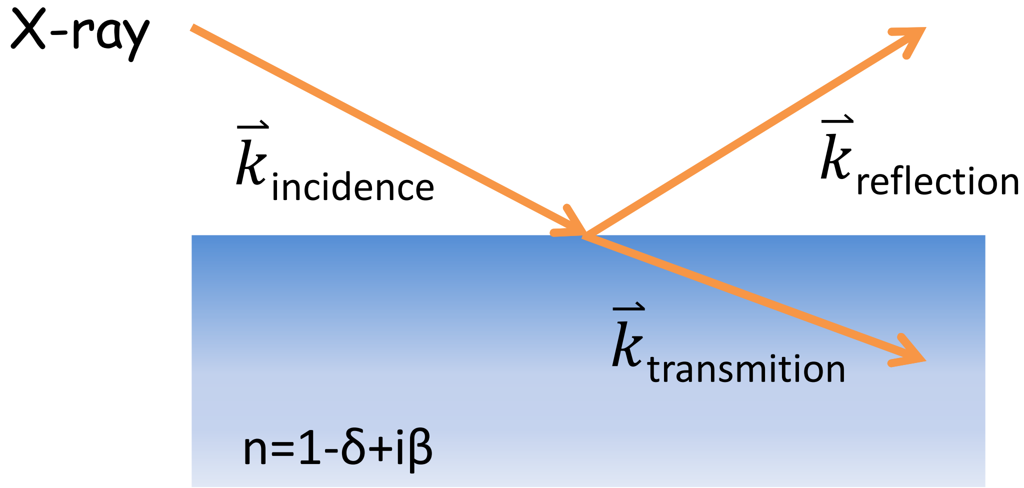
$$\text{Reflectivity} = \frac{\mathit{Reflected\ Intensity}}{\mathit{Incident\ Intensity}}$$



Yeling Dai

Group Meeting

May 27,2011

Ideal interface of two infinite mediums (static, flat, sharply terminated)

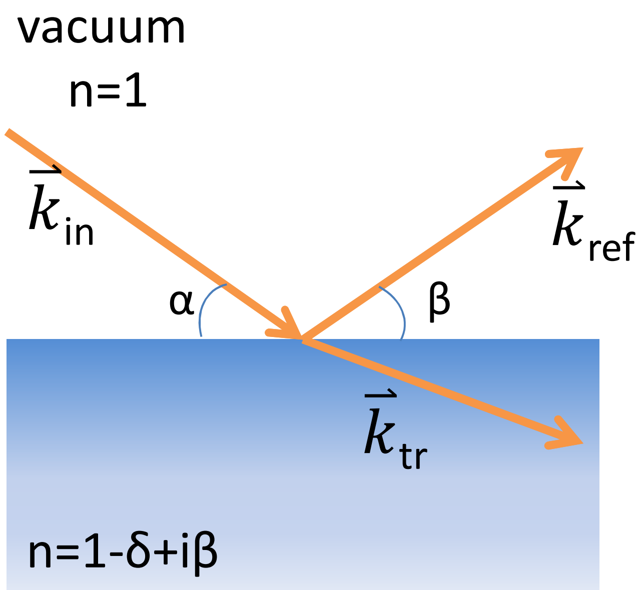
Conservation of momentum

⇓

Continuity equation of wave-function and its derivative

⇓

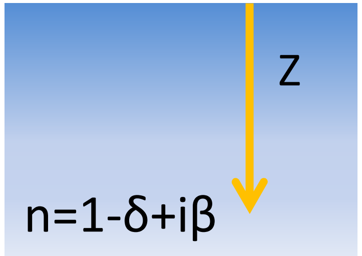
For specular reflectivity($\alpha=\beta$)



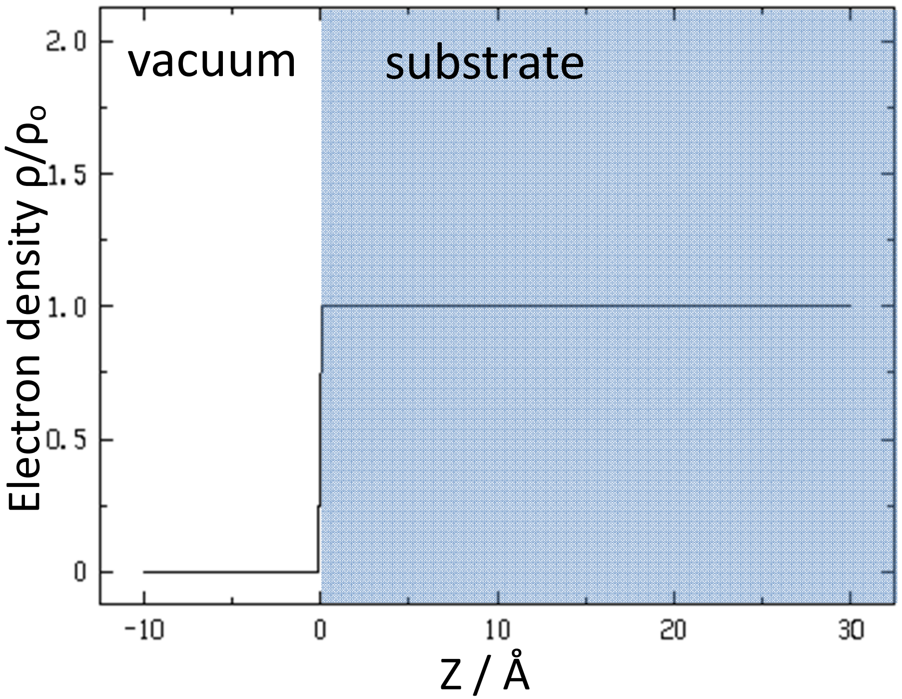
$$R_F(q_z) = \left| \frac{q_z - \sqrt{q_z^2 - q_c^2}}{q_z + \sqrt{q_z^2 - q_c^2}} \right|^2 \sim \left(\frac{q_c}{2q_z} \right)^4$$ Fresnel Reflectivity

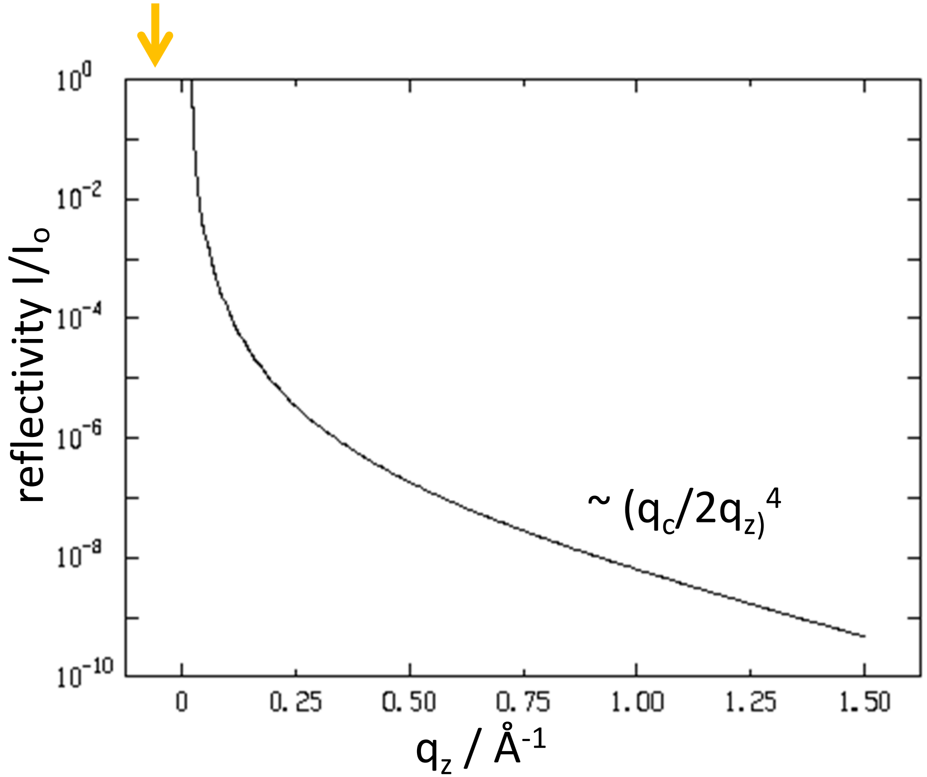
where $q_z = 2k\sin\alpha$, $q_c = 2k\sqrt{2\delta}$

Fresnel Reflectivity



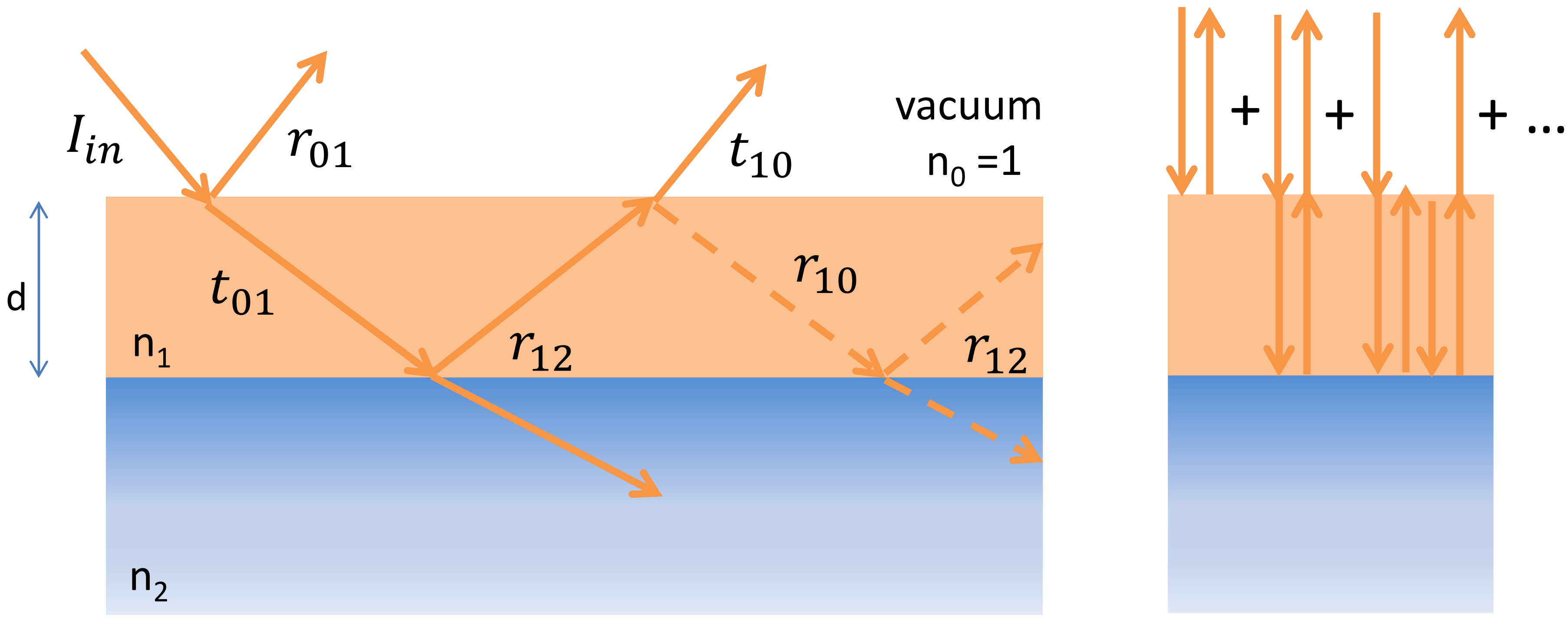
for $q_z<q_c$, $R_F=1$
(total external reflection)





$$R_F(q_z) = \left| \frac{q_z - \sqrt{{q_z}^2 - {q_c}^2}}{q_z + \sqrt{{q_z}^2 - {q_c}^2}} \right|^2 \sim \left(\frac{q_c}{2q_z} \right)^4$$

Uniform slab on the ideal substrate

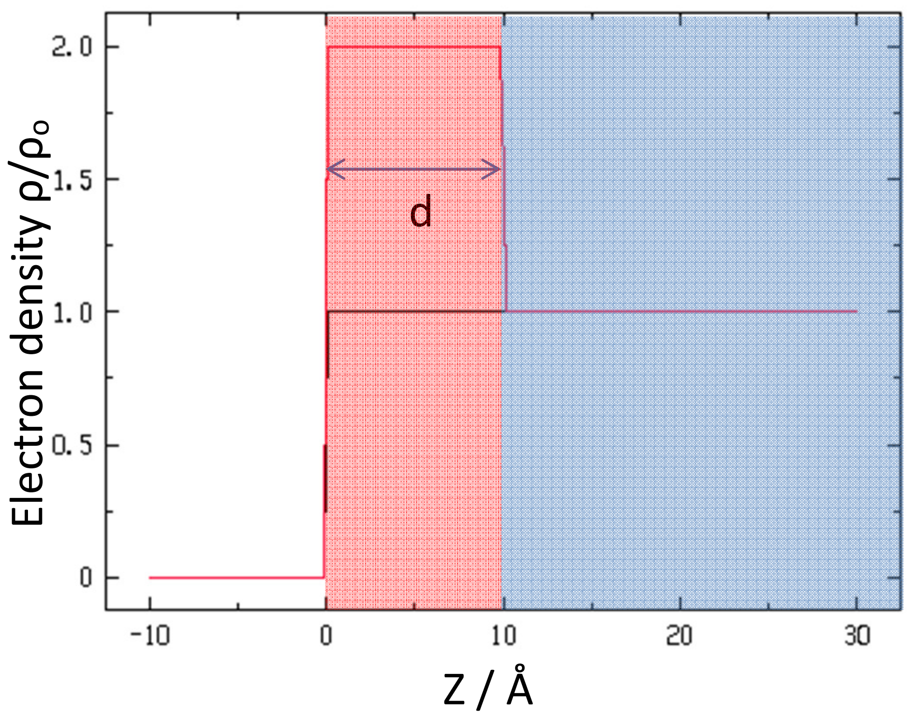


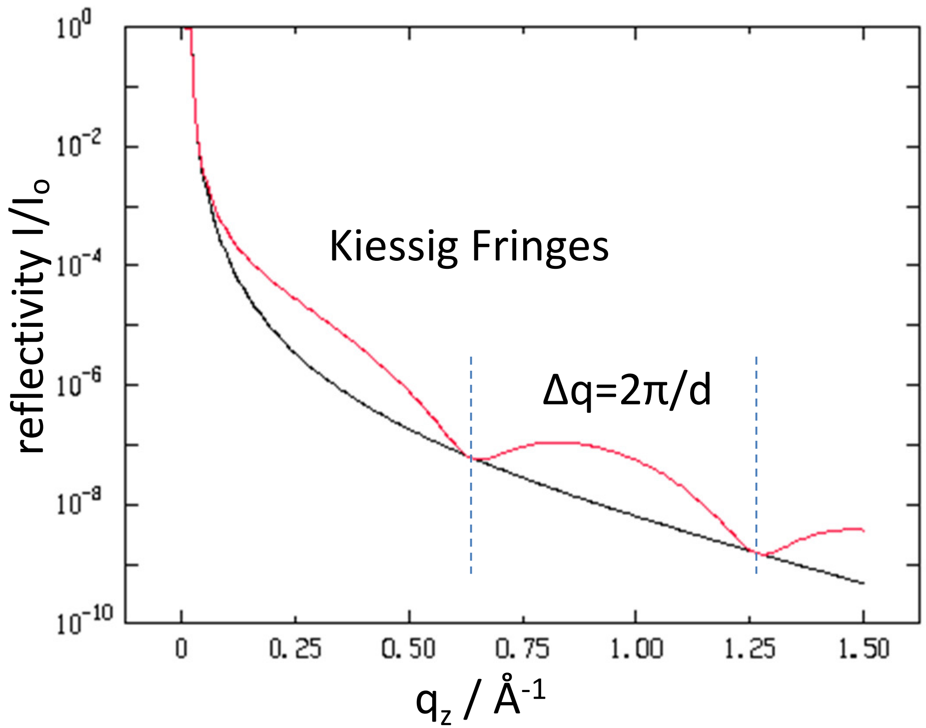
$$r_{slab}=r_{01}+t_{01}t_{10}r_{12}p^2+t_{01}t_{10}r_{10}r_{12}{}^2p^4+\ldots \;=\frac{r_{01}(1-p^2)}{1-r_{01}{}^2p^2}$$

Phase factor $p^2 = e^{iq_z d}$

$$r_{slab}\approx\left(\frac{q_c}{2q_z}\right)^2(1-e^{iq_z d}),\quad R_{slab}=|r_{slab}|^2$$

Uniform slab on the ideal substrate

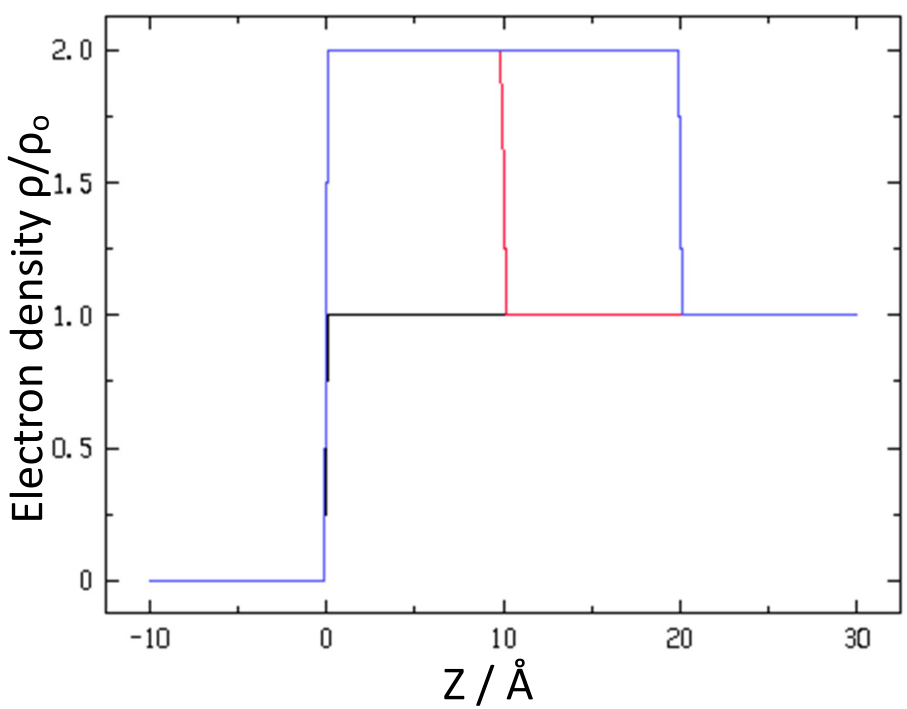


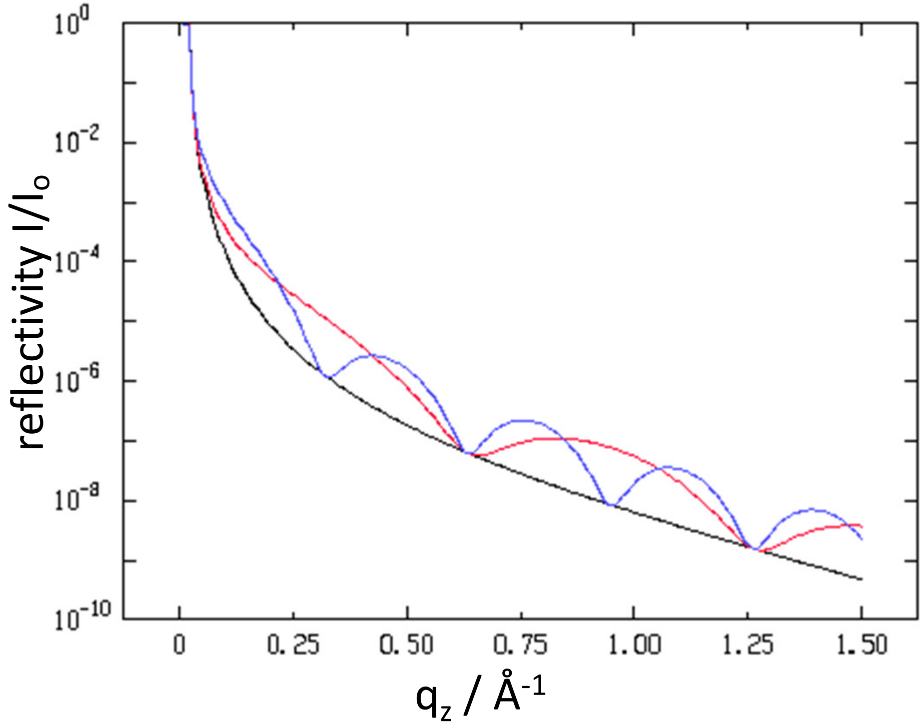


Phase factor $p^2 = e^{iq_z d}$

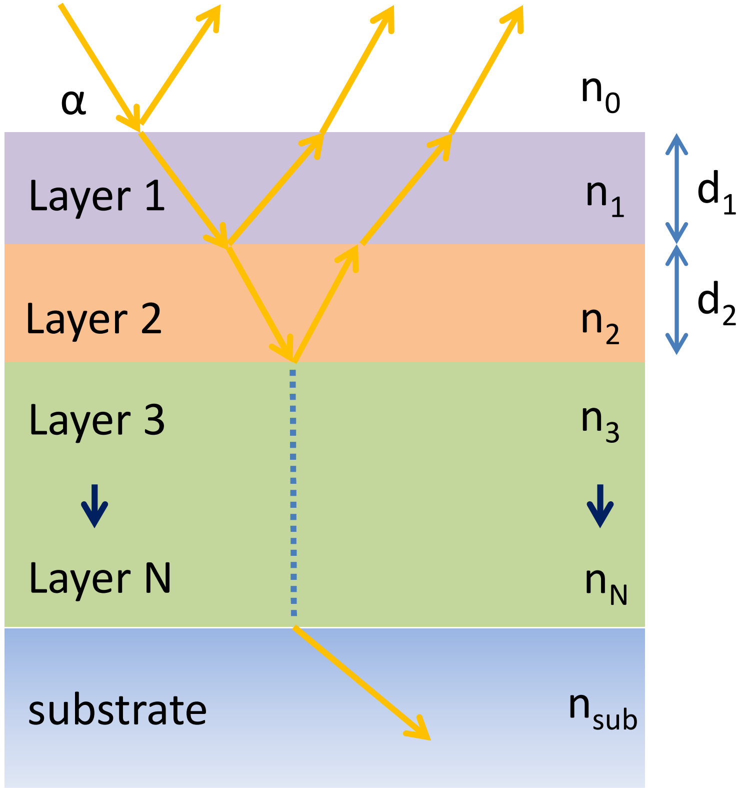
thick film → small fringes

Phase factor $p^2 = e^{iq_z d}$





Parratt Recursion for multilayers

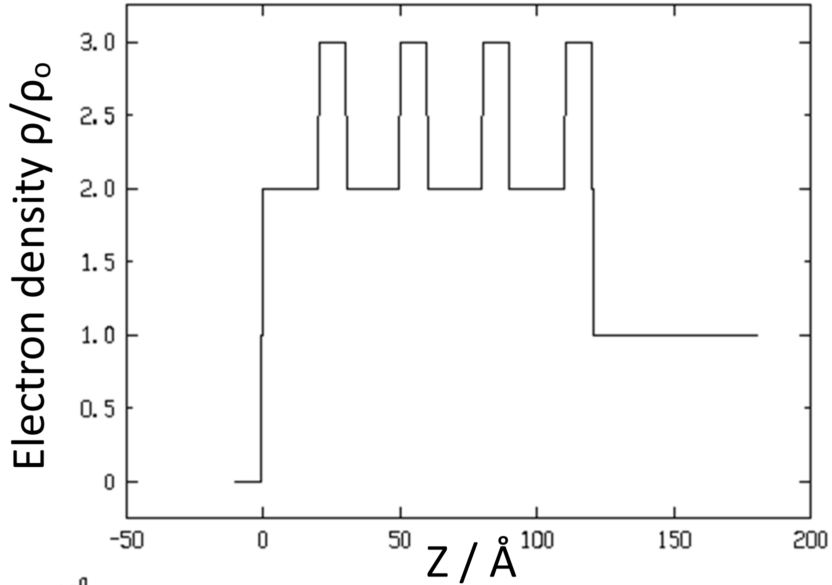


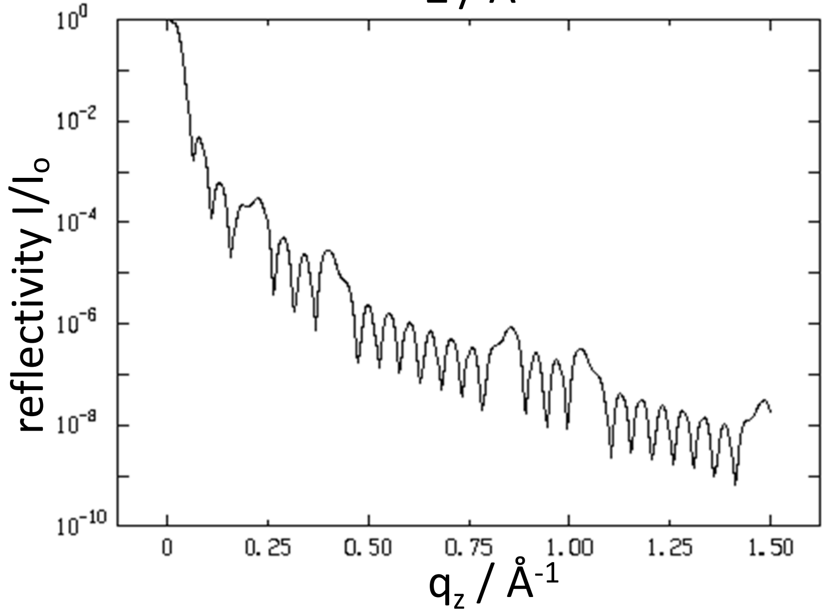
$$r'_{N,\infty} = \frac{Q_N - Q_\infty}{Qj + Q_\infty}$$

$$r_{N-1,N} = \frac{r'_{N-1,N} + r'_{N,\infty} p_N^2}{1 + r'_{N-1,N} r'_{N,\infty} p_N^2}$$

$$r_{N-2,N-1} = \frac{r'_{N-2,N-1} + r_{N-1,N} p_{N-1}^2}{1 + r'_{N-2,N-1} r_{N-1,N} p_{N-1}^2}$$

...



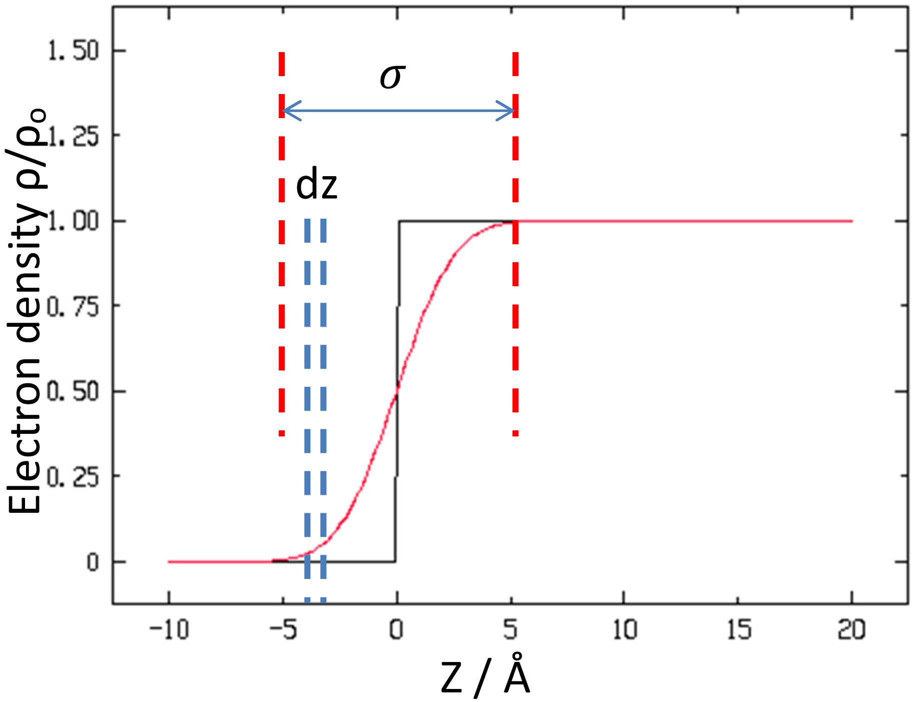


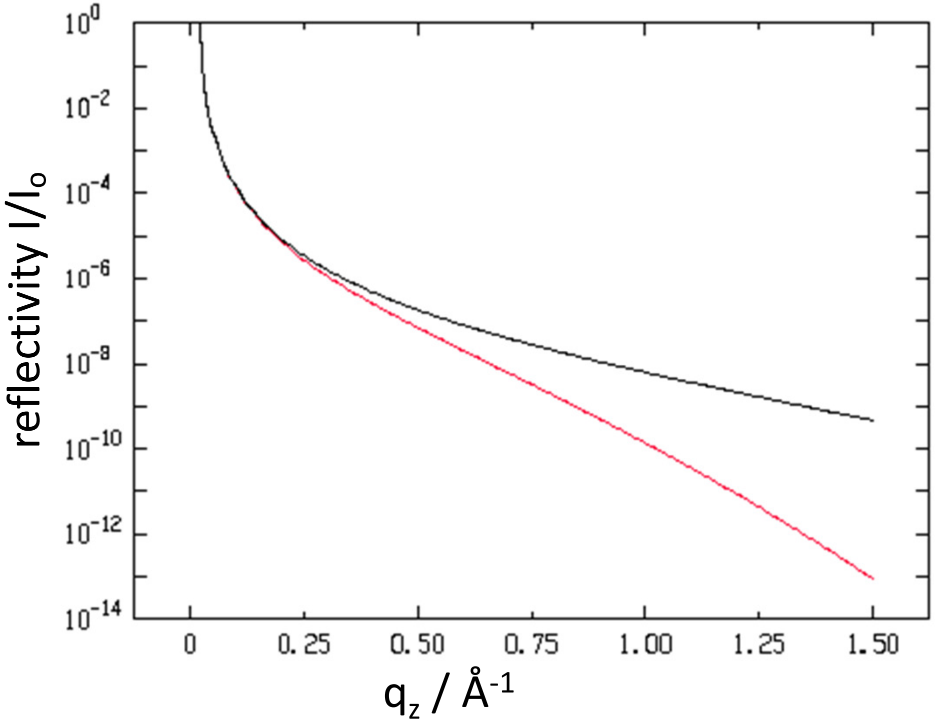
'Soft' (graded) interface
kinematical approximation(q_z>>q_c)

Master formula: $\frac{R(q_z)}{R_F(q_z)} = \left| \int_0^{\infty} \left(\frac{df}{dz}\right) e^{iq_z z} dz \right|^2$

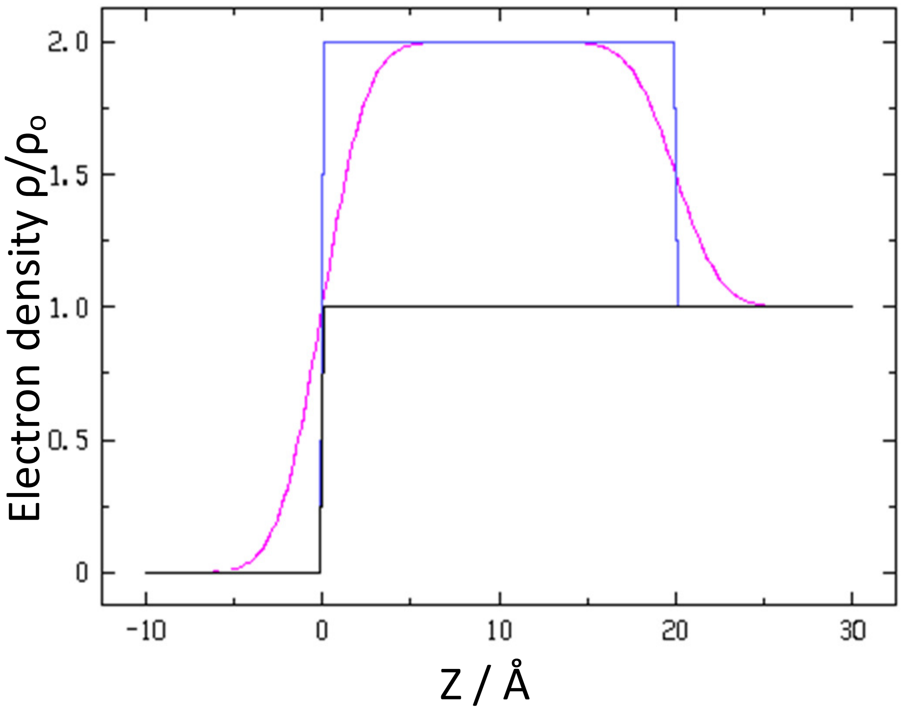
$f(z)=\mathrm{erf}\left(\frac{z}{\sqrt{2}\sigma}\right)$ ➡ $R(q_z)=R_F(q_z)\, e^{-q_z{}^2\sigma^2}$

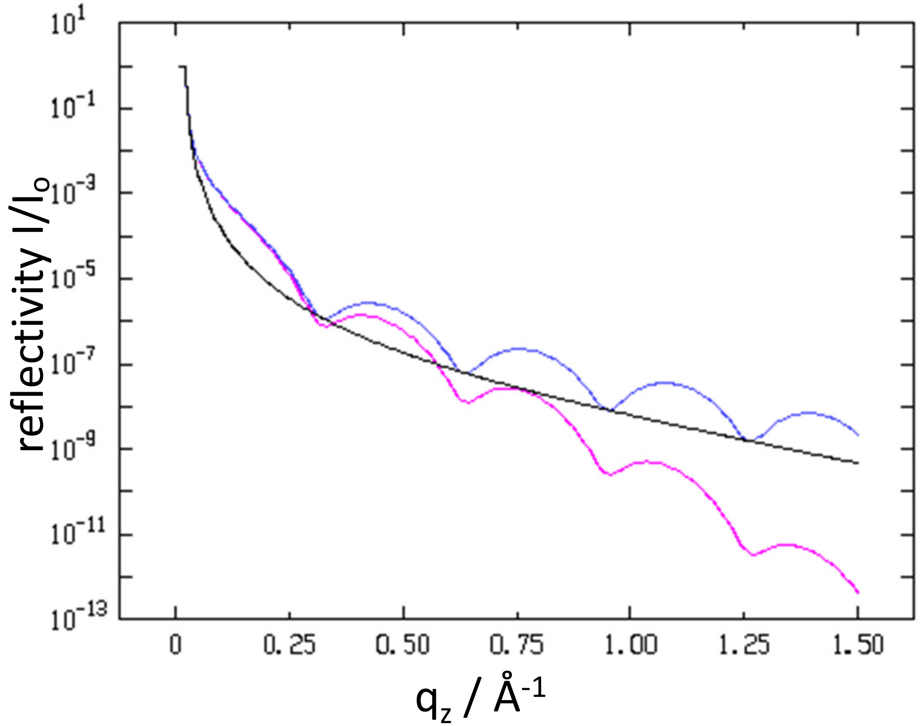
Reduce the Fresnel Reflectivity





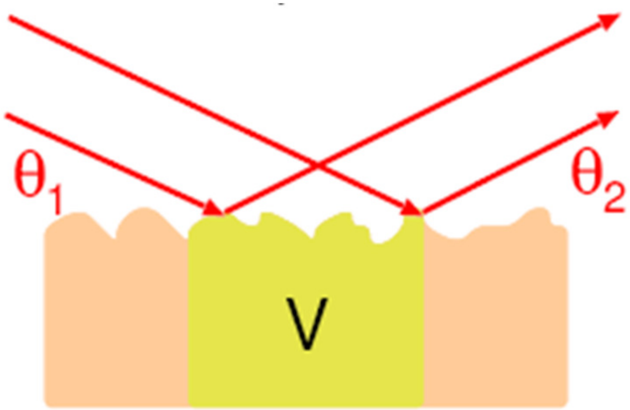
Slab with non-ideal interfaces





$$\frac{R(q_z)}{R_F(q_z)} = \left| \int_0^{\infty} \left(\frac{df}{dz}\right) e^{iq_z z} dz \right|^2 \quad \text{fit } R(q) \rightarrow f(z)$$

Rough interface



$\theta_1 \neq \theta_2$

no longer simply specular reflection but has a non-zero diffuse component (q_{xy})

Uncorrelated surface($\sigma = \sqrt{\langle h^2 \rangle}$):

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Fresnel} e^{-q_z{}^2\sigma^2}$$ height fluctuations diminish Fresnel Reflectivity

Correlated surface(h-h correlation Function C(r)):

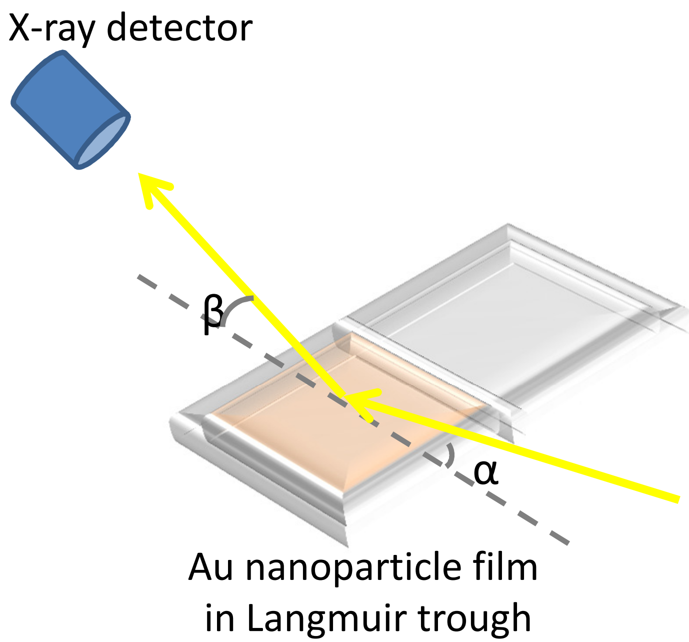
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Fresnel} e^{-q_z{}^2\sigma^2} + \left(\frac{d\sigma}{d\Omega}\right)_{diffuse}$$

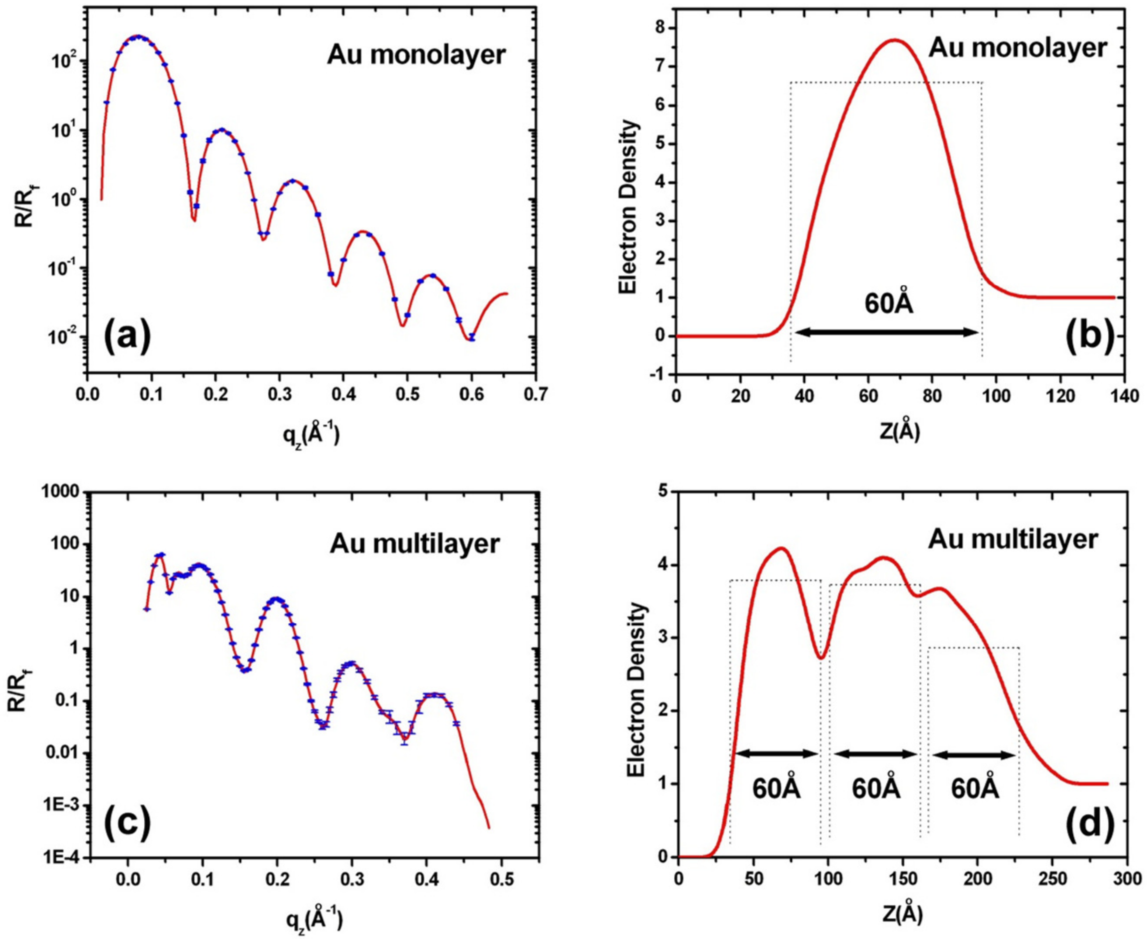
for Capyllary fluctuation:

$$C(r) = \frac{k_B T}{2\pi\gamma} K_0(q_g r)$$

Example of Reflectivity experiment

@Argonne,APS, Sector 15





Useful References:

Books:
J. Als-Nielsen and D. McMorrow "Elements of Modern X-ray Physics"
M. Tolan "X-Ray Scattering from Soft-Matter Thin Films"
Jean Daillant, Alain Gibaud "X-Ray and Neutron Reflectivity"

Theory:
L. G. Parratt,*Phys. Rev.* 95, 359 (1954)
S. K. Sinha et al., Phys. Rev. B 38, 2297 (1988)

Thank you!